JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 24, No. 1, January–February 2001

Improved Angular Observations in Geosynchronous Orbit Determination

Chris Sabol*
U.S. Air Force Research Laboratory, Kirtland Air Force Base, New Mexico 87117
and
Robert Culp†
University of Colorado, Boulder, Colorado 80309-0431

How improved angular observations can aid in the determination of satellite position and velocity in the geosynchronous orbit regime is studied. Raven, a new, automated, low-cost sensor being tested by the U.S. Air Force Research Laboratory, allows for angular observations of satellites to be made with a standard deviation of approximately 1 arc-s (which maps into approximately 170 m at geosynchronous altitude); this is an order of magnitude improvement over traditional angular observation techniques. Simulation studies are undertaken to show how these angular observations can be used in the orbit determination process both as the only tracking data source and as a supplement to other tracking data sources, such as radio transponder ranges. Parameters varied in the simulation studies include the number of observing stations, the density of the optical observations, and the number of nights of optical tracking. The studies indicate that including the improved angular observations with traditional high-accuracy range observations produces a considerable improvement in orbit determination accuracy over the range observations alone. The studies also indicate that single site geosynchronous orbit determination is an attractive alternative when combining improved angular and high-accuracy range observations.

Introduction

THE purpose of this study is to show how improved angular observations can aid the orbit determination process for geosynchronous satellites. Range-only orbit determination has traditionally been the best method of geosynchronous orbit determination due to the quality of range observations and difficulty obtaining high-accuracy angular observations, but range-only orbit determination for geosynchronous satellites has limitations due to characteristics of the range sensors and the geometry of the problem.

Obtaining quality range data for geosynchronous satellites is possible through the use of some radars and radio transponders. Range-only orbit determination is an acceptable practice for many high-accuracy orbit determination applications; however, range-only orbit determination does have its limitations, particularly for geosynchronous orbits. Because the geosynchronous satellites have little motion with respect to the Earth, the range rates of geosynchronous satellites are quite small with respect to the ground stations. This means that small errors in the range measurements result in large uncertainties in the satellite position.

To counter this, multiple tracking stations can be used to try to improve the geometry of the problem; however, the altitude of geosynchronous satellites in comparison to the radius of the Earth is quite large. Thus, even with multiple tracking stations, the geometry of the problem is still poor for resolving range-only orbit determination.

Traditionally, angular observations have been of little help. A 10 arc-s error, typical for angular observations, maps into approximately 1.7 km of error at geosynchronous altitude. Errors this large make it difficult to improve the ambiguity found in processing range data accurate to the tens of meter level. Higher angular resolution has been possible, but this usually involved methods impractical for use in operational orbit determination.

A new class of optical sensors has been devopled in the late 1990s that motivates us to reevaluate the use of astrometric techniques and the way geosynchrous orbit determination is performed. This new class of sensors is, in essence, a modern version of the Baker–Nunn camera based on small aperature telescopes made from commercial off the shelf hardware combined with computer-driven astrometric algorithms. The result is a cost-effective instrument able to take advantage of precise star catalogs to produce very accurate angular observations.

With the advent of improved optical sensors, it is possible to introduce high-accuracy angular observations into the orbit determination process as a supplement to high-quality range data. These improved angular observations help resolve the geometric ambiguity of range-only orbit determination. In addition to supplementing high-quality range data, these improved angular observations can also be used as the only data source for space surveillance and mission support roles.

Raven is a new low-cost optical telescope that can be used for deep space satellite tracking. The concept was developed by the U.S. Air Force Research Laboratory (AFRL) lasers and imaging group in Maui, Hawaii, and has been under evaluation for satellite tracking and orbit determination by the AFRL astrodynamics team in Albuquerque, New Mexico. Early calibration studies of the system have demonstrated the ability to produce topocentric right ascension and declination observations of geosynchronous satellites with noise levels under 2 arc-s (one standard deviation).[1] More recent studies have shown the noise standard deviations in the Raven observations to be closer to the 1-arc-s level.[2] It is anticipated that with refinement, the Raven will produce angular observations with less than 1 arc-s worth of noise.

Previous studies using Raven observations have shown that these improved angular observations can be valuable in geosynchronous orbit determination.[3] This study, based on the results of Ref. 2, examines the role of these improved angular observations in greater detail.

Methodology

The primary focus of this analysis is a geosynchronous satellite with a longitude of approximately 240 deg (over the California coast) and an inclination of approximately 9 deg. The satellite is supported by a system of eight ground station radio transponders that produce high-accuracy range observations for use in orbit determination at five distinct geographic locations. Similar tracking systems such as the Bilateration Ranging Transponder System used

Presented as Paper 98-4281 at the AIAA/AAS Astrodynamics Specialist Conference, Boston, MA, 10–12 August 1998; received 18 September 1998; revision received 8 June 1999; accepted for publication 25 February 2000.

*Research Aerospace Engineer, Directed Energy Directorate, 3550 Aberdeen Avenue S.E.; chris.sabol@kirtland.af.mil.

†Professor, Department of Aerospace Engineering Sciences, Colorado Center for Astrodynamics Research. Fellow AIAA.

Table 1 Radio transponder ground stations

Station number	Location	Number of passes
1	Hawaii	4
2	Hawaii	3
3	Vandenberg AFB, CA	3
4	Vandenberg AFB, CA	5
5	Colorado Springs, CO	3
6	New Hampshire	3
7	New Hampshire	2
8	Thule, Greenland	4

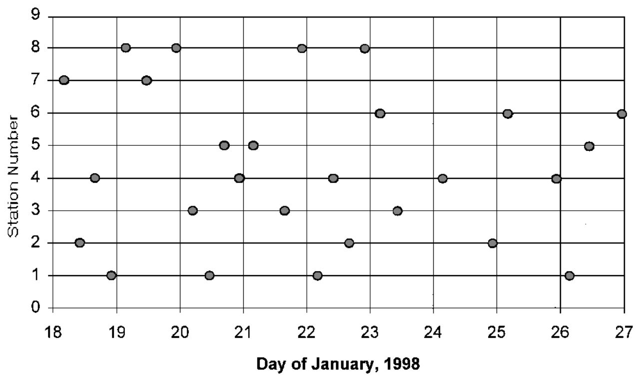

Fig. 1 Range observation distribution.

by NASA to support TDRSS are reported to have noise levels on the order of 2–3 m (one standard deviation).[4,5] Table 1 describes the stations and the amount of data obtained from each station over the time period of 18–27 January 1998.

Each pass of tracking from each station collected 30 range observations; this resulted in a total of 810 range observations from the 27 passes over the 9-day period of study. The length of each pass never exceeded 30 min and was sometimes as short as 7 min. Figure 1 plots the temporal distribution of the observations.

The first part of the analysis focused on understanding the limitations of range-only orbit determination solutions for this satellite through a combination of real data and simulated data studies. Next, simulation studies using high-accuracy topocentric right ascension and declination observations to supplement the radio transponder ranges in the orbit determination process were conducted. While the number of range observations was kept constant, the studies varied the number of optical sensors, the density of the angular observations, and the number of nights of optical tracking. The results of these studies demonstrate the utility of these improved angular observations for a satellite with a large range-based tracking network.

Further simulation studies were performed limiting the number of available range observing stations. These scenarios included single-site orbit determination using high-accuracy range and angular observations as well as angles-only scenarios. These studies demonstrate how improved optical sensors can help produce quality orbit solutions without an extensive tracking network.

Studies involving the use of improved angular observations for space surveillance-type orbit determination are available in Ref. 2.

The analysis tool used in all studies presented here is the operational version of the NASA Goddard Space Flight Center Trajectory Determination System (Ops GTDS).[6]

Data Simulation Description and Validation

Because angular data were not available for this study due to sensor calibration issues, data simulations were used to investigate the impact these observations would have on the geosynchronous orbit determination problem. To ensure the integrity of the data simulations, comparisons were made between real and simulated range-only cases. If the simulated data fits were similar to the real data cases, it would build confidence in the results of the simulated cases. Then conclusions could be drawn about the utility of the improved angular observations from the simulations.

Table 2 Orbit generator models

Property	Method
Integrator	Cowell special perturbations
Step size	400 s
Geopotential	4×4 JGM2
Third body	Solar, lunar point masses (JPL DE 118,200)
Solar radiation pressure	Spherical body, single reflectivity constant Cr
Empirical acceleration	Constant along track

Table 3 Real data range bias differences

Station number	Estimated − reported, m
1	−11.26
2	−16.70
3	0.75
4	−0.10
5	7.34
6	13.75
7	37.06
8	42.53

First, fits were made to the 9 days of radio transponder range observations shown in Table 1 and Fig. 1. Table 2 shows the force modeling used in the Ops GTDS differential correction program's weighted least-squares batch estimator. The estimated parameters included the satellite position and velocity, a single reflectivity constant for solar radiation pressure, and a constant along-track acceleration over the entire fit span. The along-track acceleration was required to account for daily momentum dumps performed by the satellite.

Two real data cases were investigated. The first orbit solution, case 1, estimated range biases from all radio transponder ground stations. The second solution, case 2, did not estimate the range biases but used the reported bias values provided by the satellite operator from each sensor. The two orbit solutions were then compared over the 9-day fit span at 20-min intervals.

The comparisons, case 1 minus case 2, showed periodic differences in the radial direction (24-m amplitude, 48 m from peak to peak) and cross-track direction (293-m amplitude) and an along-track drift (from −63 to −373 m) with periodics (on the order of 40 m). These are sizable differences that can be attributed to the difference in range station bias values. Table 3 shows the difference between the reported range station bias values used in case 2 and the estimated values obtained from case 1.

Part of the bias differences in Table 3 can be attributed to atmospheric effects. The data processing did not include atmospheric effects, and the differences have some elevation angle dependence. However, there do appear to be differences between the reported values and the estimated values aside from atmospheric effects. Consider stations 6 and 7: These stations are collocated so the atmospheric effects should be similar for both. Yet the bias differences are on the order of 20 m. For stations 1 and 2, the difference is over 5 m. Either the calibration for these stations is poor or the bias solutions are poor. Note that maintaining calibration of these radio transponder sensors is nontrivial considering the bias drift in the sensors and the coupling with the satellite radio transponder delays.

Next, a data simulation was set up to replicate the real range data scenario. The stations and tracking patterns (passes and density) were represented as closely as possible to reflect the real world conditions. Noise was added to the simulated data based on the observation standard deviation reported from the real data cases. Biases were added to all stations based on the real data case 1 range bias solutions. The case 1 real data solution was used as the truth orbit from which the simulated observations were constructed.

Fits were then performed to the simulated data: one simulated case, simcase 1, in which all range station biases were estimated and another case, simcase 2, in which the reported bias values (different from the truth biases by the values listed in Table 3) were used. The a priori bias values for both cases were the reported bias values (unless otherwise noted, the rest of the data simulations used the truth values as a priori guesses to speed convergence). The simcase solutions were then compared to the truth orbit used to construct the simulated observations.

Comparing simcase 2 to the truth orbit showed periodic differences in the radial direction (24-m amplitude, 48 m from peak to peak) and cross-track direction (299-m amplitude) and an along-track drift (from −62 to −361 m) with small periodics (on the order of 40 m). This confirms the notion that range errors, in the form of biases for this case, lead to large position errors in geosynchronous orbit determination.

Table 4 lists the orbit differences between the truth orbit and real data solution, case 2, and simulated data solution, simcase 2. Table 4 shows that the data simulations are representative of the real world conditions. Figure 2 summarizes this data simulation verification/validation procedure.

Note that orbital accuracy from real data studies is usually inferred from solution overlaps: If the same bias errors are present in the overlapping real data solutions, that is, if the reported bias values are used in both fits, the along-track biases will effectively cancel in the overlap comparison leaving the analyst unaware of the large errors.

Comparing simcase 1 to the truth orbit showed periodic differences in the radial direction (0.6-m amplitude, 1.2 m from peak to peak) and cross-track direction (1.3-m amplitude) and an along-track drift (from −31 to −35 m) with small periodics (on the order of 1 m). Table 5 shows the difference between the simcase 1 range station bias solutions and the truth values used in the simulated observations.

Table 5 shows bias solution errors below 3.5 m. These results appear to be consistent with the real data cases and show that the biases can be estimated given the amount of data available for this satellite. However, the amount of error remaining in the range stations bias solutions does lead to an along-track bias of 30 m.

Table 4 Real data and simulated data (simdata) comparisons

	Difference from simcase 1, truth	
Property	Real data case 2, m	Simdata simcase 2, m
Radial periodics	24	24
Cross-track periodics	293	299
Along-track minimum, maximum	−63, −373	−62, −361

Table 5 Simdata range bias differences

Station number	Estimated − truth, m
1	3.26
2	3.42
3	0.06
4	0.05
5	−1.33
6	−2.79
7	−3.21
8	−1.01

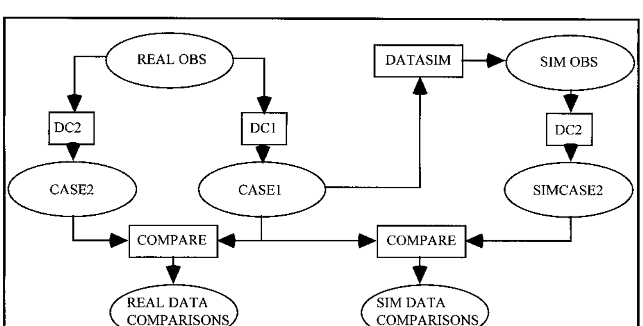

Fig. 2 Data simulation validation flow chart.

Analysis and Results

This section is divided into three main subsections: range only, extensive range and angles, and limited range and angles.

Range Only

In the range-only studies, an effort is made to understand the limitations of the range-only geosynchronous orbit determination solution given an extensive tracking system.

Table 3 shows that there are large differences between the reported bias values and the estimated bias values for all of the range stations. In simcase 1, it was shown that the biases could be estimated to the 3.5-m level (recall Table 5).

Assuming that the real data case 1 bias solutions for stations 6 and 7 are accurate to 3.5 m and that the differences between the bias solutions and the reported values are on the order of 10–20 m, one must then conclude that the range station calibration bias values cannot be much more accurate than 10–20 m. The calibration must be on the several meter level (better for some stations, worse for others). The simcase 1 results showed that bias errors on the 3-m level lead to a 30-m along-track error in the orbit solution. It would appear that the reported bias values have errors larger than 3 m and that using them results in considerably larger orbital errors (recall simcase 2).

Variable Fit Span

Studies were performed to see if the bias solutions could be improved by changing the amount of data used in the fits. If the fit span is shortened to below 9 days, the bias solutions worsen; there were not enough data to solve for the biases effectively. If the fit span is lengthened to 13 days, the bias solutions do not improve; this can be partly attributed to the uncertainty in the dynamic models over longer periods of time. Table 6 shows the real data bias solution standard deviations for the 9- and 13-day fit spans. The 9-day case corresponds to case 1. The standard deviations for the 9-day fit are fairly consistent with the errors in the simulated data bias solutions, simcase 1 (Table 5). The differences in the bias solutions are on the same order as the standard deviations. When estimating station biases, there will always be some uncertainty in the solutions.

Lower Inclinations

Next, the effect of lowering the geosynchronous orbit's inclination on the bias solutions was investigated. Range-only simulation studies were performed for satellites at approximately 9- (as in simcase 1), 4-, and 0-deg inclinations. The other orbital elements (semimajor axis, eccentricity, etc.) were kept the same for the three cases. The results show that observability of the range station biases is lost as the inclination is lowered. This stems from that the range rate of a theoretically geostationary satellite is zero and that any satellite in the geostationary ring could produce the observed ranges given an arbitrary bias term. With less observability, the bias solutions get worse. As the bias solutions get worse, the orbit solutions get worse. Table 7 shows the bias solution errors for the three different inclination cases. Table 8 shows the orbital errors. The 9-deg inclination case differs slightly from simcase 1 because slightly different bias values and initial conditions were used. For the 0-deg inclination case, station 8 was not visible, and all of the station 6 observations were edited out in the differential correction process.

Table 6 Real range data bias solution standard deviations

Station	9-Day fit, m	13-Day fit, m
1	7.89	8.42
2	7.91	8.51
3	0.18	0.14
4	0.16	0.14
5	2.87	3.09
6	7.23	7.76
7	7.22	7.75
8	2.26	2.41

Table 7 Bias errors for range-only inclination cases

Station number	i = 9 deg, m	i = 4 deg, m	i = 0 deg, m
1	2.91	7.78	529.82
2	3.06	7.92	530.33
3	0.07	0.13	6.41
4	0.05	0.09	6.52
5	−1.09	−2.86	428.00
6	−2.47	−6.92	N/A
7	−2.88	−7.33	−482.86
8	−0.93	−2.31	N/A

Table 8 Orbit errors for range-only inclination cases

Difference from truth	i = 9 deg, m	i = 4 deg, m	i = 0 deg, m
Radial periodics	0.6	0.6	1.8
Cross-track periodics	1.7	1.9	127
Along-track minimum, maximum	−27, −31	−78, −81	−5444, −5457

Table 9 Single station bias and resulting along-track errors

Station 5 bias error, m	Along-track bias, m
0	0
1	29.5
5	144
10	285

Station Bias not Estimated

Clearly, one cannot solve for all biases at lower inclination orbits with only range data. The next step would be to solve for all station biases except one. Simulation studies indicated that it is unwise to choose a station that was at the same longitude as the satellite when deciding which bias not to solve for. Doing so results in larger errors. For these studies, it was decided to not solve for the bias in station 5.

Fits to the simulated observations for the original 9-deg inclination orbit were generated for cases having 0-, 1-, 5-, and 10-m errors in the station 5 bias. The rest of the station biases were estimated. Table 9 shows the relationship between the error in the station 5 bias and the resulting along-track orbit error. The trend appears linear. Additional studies to determine this relationship mathematically and to generalize for other stations and satellites were not undertaken.

Recall Table 1 to see that the difference between the estimated station bias and the reported value was 7.34 m, which would result in an long track error of over 200 m. If the calibrated value of the range bias were 1 m, the along-track bias would still be on the order of 30 m.

Summary

From the range-only studies, it was observed that the reported bias values of the range stations are accurate to the several-meter level at best. All range biases can be estimated only for inclined orbits, and that still leaves up to several meters of uncertainty in the solutions. Not choosing to solve for one or more biases is limited by the error in the unestimated biases, which can lead to fairly large orbital errors. Under all circumstances it was found that small errors in the range observations lead to significant along-track errors in the orbit solutions. Thus, the range-only solutions are limited by the accuracy of the range sensor calibration or the ability to solve for the sensor biases. If the biases are very well known, the range-only solution has proven to be quite accurate; however, for the 9-deg inclination case presented, the calibration is poor, and estimating biases has proven to deliver orbits only to the 30-m level and possibly much worse for lower inclination cases.

Figure 3 shows orbital errors for the best range-only orbit solution where all biases were estimated.

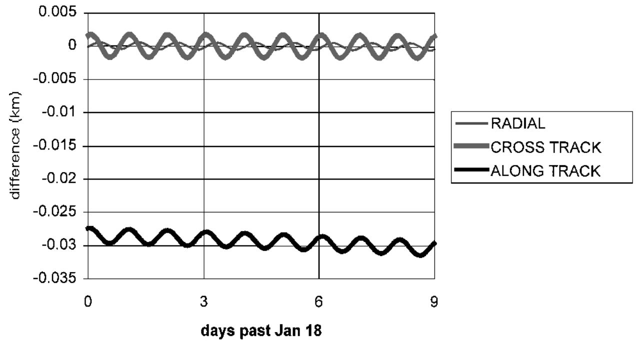

Fig. 3 Best range-only orbit solution errors.

Extensive Range and Angles

After recognizing the limitations of range-only geosynchronous orbit determination, improved angular observations were introduced to the simulations studies.

In all of the simulation cases involving angular data, 1 arc-s of noise (one standard deviation) was added to the right ascension observations, and 0.5 arc-s of noise was added to the declination observations. This is a reasonable assumption based on current Raven data processing and the improvements expected in the near future from new star catalogs and refined endpoint detection. No biases were added to the angular observations. This assumption is based on the ability of optical sensors to calibrate themselves using well-known objects such as reference stars or global positioning system (GPS) satellites.

The primary range and angles scenario included the same range stations and observations as the primary range-only case (reflecting the real world tracking for this satellite) and two optical sensors. The optical sensors were located in Albuquerque, New Mexico, and Maui, Hawaii, also the location of two Raven telescopes. Some studies included a third optical site at Vandenderg Air Force Base California, collocated with range station 3.

It was assumed that the sensors could track 6 out of the 9 nights of the fit span. This is a reasonable assumption based on the history of the Raven sites. Actually, the Raven telescope in Albuquerque was able to track the satellite 8 out of the 9 nights during this particular fit span. Unfortunately, the observations were not available for this study due to sensor calibration problems. Each sensor tracked for 8 h each night. For most of the simulated cases, the tracking was distributed such that the Maui station tracked on the nights of 18–23 January and the Albuquerque station tracked on 20 January and 22–26 January. The simulations also assume the optical sensors could record observations at a rate of one observation pair (right ascension and declination) per minute. This roughly reflects the data collection ability of the current Raven sensors.

Like the real and simulated data cases described in the data simulation validation, the estimated parameters in the differential correction process included position, velocity, reflectivity constant, along-track acceleration, and all range biases. In all cases the differential correction process weighted the observations according to the amount of noise added in the data simulation.

The primary range and angles case described earlier (2 optical sensors, 6 nights tracking from each, 8 h per night, 1 observation pair per minute) reflects a scenario in which two telescopes are dedicated to tracking the geosynchronous satellite. Other cases are variations of this scenario in which parameters were varied, such as the number of optical tracking stations, nights of optical tracking, and density of the angular observations.

The primary data simulation case produced 2886 observation pairs from each optical station for a total of 11,544 angular observations, compared to 827 range observations from the eight simulated radio transponders.

Fitting to the range and angles data produced outstanding results. Table 10 contains the bias estimation errors, Table 11 provides the orbital errors for the range and angles case in comparison to the best range-only case, and Fig. 4 plots the orbital errors for the range and angles case over time.

Table 10 Range and angles range bias errors

Station number	Estimated − truth, m
1	0.21
2	−0.11
3	0.03
4	−0.04
5	−0.27
6	0.31
7	0.05
8	0.22

Table 11 Range and range and angles orbit errors

	Difference from truth	
Property	Range only, m	Range and angles, m
Radial periodics	0.6	0.2
Cross-track periodics	1.7	1.9
Along-track minimum, maximum	−27, −31	0, 2.5

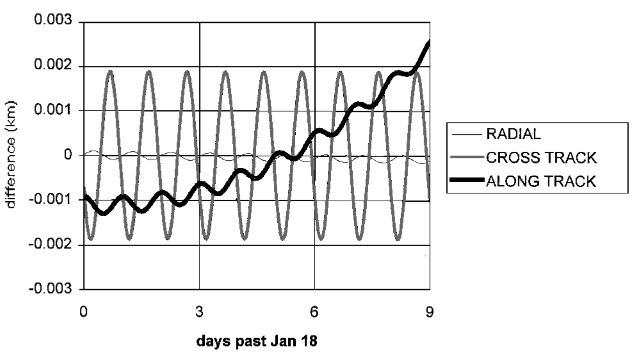

Fig. 4 Primary range and angles orbit solution errors.

Figure 4 shows the along-track bias due to the uncertainty in the bias solutions is gone. The bias solutions themselves are an order of magnitude better than in the range-only case. It appears that the range and angles solution has difficulty in capturing the along-track acceleration. This results in a small drift in radial error and a parabolic runoff in the along-track direction. Closer study of the range-only cases shows a similar along-track runoff masked by the scale of the along-track errors.

The range and angles orbit solution is more accurate than the range-only case. The range and angles case did, however, include two more viewing stations (only one was geographically distinct) and an additional 11,544 observations obtained from long, dense passes. To demonstrate that the angular observations do, in fact, bring additional, useful information to the orbit determination process, a range-only case replicating the primary range and angles tracking scenario was created. Here, the optical sensors in Albuquerque and Maui were designated as radio transponders collecting range. The tracking schedule was kept identical to the range and angles case. A bias was added to the new range stations because biases are a real world characteristic of these sensors and, as demonstrated earlier, calibration can be poor. The new biases were included as estimated parameters in the differential correction.

The fit to the 10-station range case showed improvement over the 8-station range-only case, but bias uncertainties and along-track errors remained. The orbit solution had 0.4-m (0.8 m peak to peak) radial periodics, 5.2-m cross-track periodics, and an along-track bias of approximately 20 m. Clearly, the angular observations provide a different type of information that is very valuable to the geosynchronous orbit determination problem.

Note that the 10-station range case contained fewer observations than the range and angles case because the Albuquerque and Maui stations produced one range observation every minute of tracking instead of two angular observations. This does not impact the conclusions, however, because the angular observation density can be decreased to one observation pair every 2 min and a similar level of accuracy is maintained (to be discussed later in this section).

Once it was determined that the improved angular observations could help improve geosynchronous orbit determination accuracy, studies were performed to investigate how much angular data is required to make significant improvements.

Number of Optical Sensors

The first parameter considered was the number of optical stations. Several scenarios were studied where the number of observations (total nights tracking and observation density) were kept equal while the number of viewing stations was varied between 1 and 3. Table 12 shows the orbit errors for the three most telling cases (12 total nights tracking, 1 observation pair/minute). Here the distribution was kept identical across the three cases. For the 1-station case, the Maui station coordinates were changed to coincide with the Albuquerque station; for the 3-station case, two nights of tracking from Albuquerque and Maui were transferred to a station at Vandenberg Air Force Base.

Table 12 shows little variation from 1 to 3 stations. Figure 5 shows the along-track bias as a function of the number of optical sensors for several different tracking scenarios. Note that the observation distribution varies between some of the cases, and this affects the results. There is also some variance from case to case that may be on the order of a several meters. In general, it appears that the number and location of the viewing stations have little impact on the orbit accuracy with all other things (observation distribution and number) being equal.

Optical Observation Density

After varying the number of optical viewing stations, several simulation cases were performed varying the density of the angular observations. In addition to the primary case density of one observation pair per 1 min, data simulations generated cases where the angular observations were collected at the rate of one observation pair per 2, 5, 10, 20, and 40 min. Three scenarios were considered: two optical stations tracking for 6 nights each (12 nights total), two optical stations tracking for 4 nights each, and three optical stations tracking for 6 nights each. Table 13 shows the along-track bias for

Table 12 Station dependence on orbital errors

	Optical stations		
Property	1	2	3
Radial periodics, m	0.2	0.1	0.2
Cross-track periodics, m	1.9	1.5	1.4
Along-track minimum, maximum, m	0, 2.5	0, 3.1	0, 3.6

Table 13 Along-track error as a function of optical density

Density, min/ observation	3 Stations, 18 nights, m	2 Stations, 12 nights, m	2 Stations, 8 nights, m
1	1.4	1.2	0.6
2	1.4	2.5	3.4
5	7.5	8.0	3.0
10	13.7	7.4	6.9
15	2.3	5.0	18.5
20	2.9	17.4	16.6
40	15.8	21.2	N/A

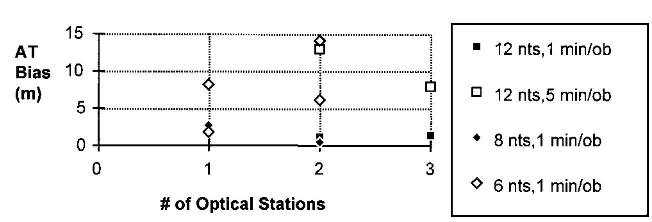

Fig. 5 Along-track accuracy as a function of optical station number.

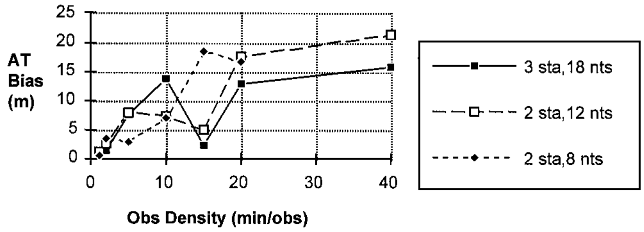

Fig. 6 Along-track accuracy as a function of optical data density.

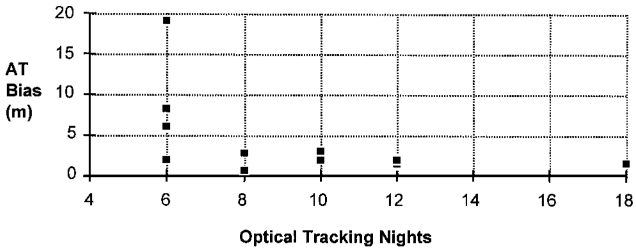

Fig. 7 Along-track errors as a function of nights of optical tracking.

all of the cases, and Fig. 6 plots the information. In both Table 13 and Fig. 6, the along-track bias is taken as the mean of the absolute minimum and maximum along-track errors.

Both Table 13 and Fig. 6 show a fair amount of uncertainty in the solutions for the 5–15 min per observation density results. Figure 6 does indicate a general trend for the along-track error as a function of observation density, however. It appears that for all of the cases examined, along-track biases are present in the solutions with optical observations densities less than 2 min per observation pair. It also appears that the error/density relationship is a function of the number of nights of tracking; the 18-nights tracking case fairs better than the 12- and 8-nights tracking cases. This dependence could be linked to the total number of observations or the distribution of the observations.

Nights Tracking Optically

Some links of orbit accuracy to the number of nights of optical tracking have already been observed. The next data simulation cases were performed to investigate the dependence of orbit accuracy on the number of nights of optical tracking. The scenarios varying the number of nights tracking optically included several different data distributions; even with an equal number of observations, different data distributions produce varying orbit accuracies. In practice, weather conditions dictate the number of nights tracking and observation distribution from the optical sensors; one should not expect to have more than 6 out of 9 nights of clear tracking for the optical sensors on average. These studies try to understand, in a general sense, how the orbit accuracy degrades with fewer nights tracking.

Simulations were conducted ranging from 1 to 3 optical stations, over 6, 8, 10, 12, and 18 nights with an observation density of one observation pair per minute. Figure 7 shows the along-track errors as a function of the number of nights of optical tracking. Again, the along-track bias is taken as the mean of the absolute minimum and absolute maximum along-track errors. Figure 7 shows that along-track errors become prevalent below 8 nights of optical tracking. If it is assumed 1 station can track 6 out of 9 nights, then one must conclude that two optical sensors are required to eliminate the along-track errors. Further studies modeled 1 and 2 optical stations with an observation density of 10 min per observation pair for 6, 8, 10, and 12 days with similar conclusions.

Lower Inclinations

For the range-only case, it was observed that it becomes more and more difficult to estimate the range station biases as the geosynchronous orbit inclination decreases. For the 9-deg inclination case, the biases could be effectively estimated to a greater accuracy level than the apparent accuracy of the reported bias values. For the 0-deg inclination case, the bias solutions were terrible, limiting the range-only case to the accuracy of the reported bias values.

Table 14 Orbit errors for range and angles inclination cases

	Difference from truth, m		
Property	i = 9 deg	i = 4 deg	i = 0 deg
Radial periodics	0.2	0.1	0.3
Cross-track periodics	1.5	1.6	2.3
Along-track minimum, maximum	0, 2.9	0, 2.7	0, 2.1

For the 9-deg inclination case, it was shown that supplementing the range observations with improved angular observations allows for the estimation of all bias values far more effectively and reduces all resulting orbital errors to the meter level. With the lower inclination cases, similar results were produced. Table 14 contains the orbital errors for 0-, 4-, and 9-deg inclination cases using the primary range and angles tracking scenario (2 optical stations, 12 nights tracking, one observation pair per minute). Compare the values in Table 14 to the range-only values in Table 8.

Shorter Fit Spans

The additional volume of angular observations allows the fit span to be reduced and still accurately solve for all range station biases. For the range-only case, at least 9 days of tracking were needed to estimate effectively the biases, and the orbit still contained 30 m of error. For the primary range and angles case, reducing the fit span from 9 to 5 days did not reduce accuracy at all. This could be valuable for satellites where the dynamics are difficult to model over longer fit spans or for postmaneuver orbit determination.

Summary

In the extensive range and angles cases, it is shown that improved angular observations help resolve the range bias ambiguity present in the range-only cases. With the angular observations, all range biases can be effectively estimated; this includes low-inclination cases.

The studies show that 8 nights of optical tracking is desirable over the 9-day fit span. This implies that 2 optical stations are needed to produce high-accuracy orbits. However, the desired density (2 min per observation pair) allows the tracking sites to track two satellites. Thus, two satellites could conceivably share two optical sensors with better results than each satellite monopolizing single optical sensors.

Limited Range and Angles

Up to this point, all of the data simulations modeled an extensive range tracking network: 8 stations at 5 distinct geographic locations spanning from Hawaii to Thule, Greenland, to New Hampshire. In this subsection, scenarios are investigated where the range tracking network is far more limited. Efforts are made to determine if the optical observations allow the number of radio transponder range stations to be reduced or even eliminated (angles-only orbit determination). The 9-deg inclination case was used for all studies.

Range Only

The first studies investigate how reducing the number of tracking stations affects orbit accuracy for the range-only case. The primary range-only case consisted of 8 range stations at 5 distinct locations. A data simulation case was constructed to reduce the number of stations to 5; the three pairs of collocated stations were combined into 1 station. Another simulation reduced the number of stations to 3: station 2 in New Hampshire, station 3 in Vandenberg Air Force Base, California and station 5 in Colorado Springs, Colorado. The remaining station tracking passes were distributed among the 3 stations. In all range-only cases, the total number of observations were kept roughly equal. Biases were modeled and estimated for all stations.

Table 15 shows the orbital errors for the 8-, 5-, and 3-station range-only cases. As the number of range stations is reduced, the orbital errors increase.

Single-site range-only orbit determination is not possible for geosynchronous satellites when the satellite is at the same longitude as the tracking station, and it is very difficult and undesirable when the station is at a different longitude.

Table 15 Limited range-only orbital errors

	Difference from truth, m		
Property	8 Stations	5 Stations	3 Stations
Radial periodics	0.6	0.6	0.4
Cross-track periodics	1.7	9	1
Along-track minimum, maximum	−27, −31	−31, −33	−97, −100

Table 16 Three range station plus angles orbit errors

	Difference from truth, m		
Property	Case 3a	Case 3b	Case 3c
Radial periodics	0.2	0.4	0.7
Cross-track periodics	1.9	1.1	5.5
Along-track minimum, maximum	1.2, 2.5	0, 5.1	0, 6.0

Table 17 Single-site range and angles orbit errors

	Difference from truth, m		
Property	Case 4a	Case 4b	Case 4c
Radial periodics	4.8	1.0	0.5
Cross-track periodics	5.8	9.2	1.3
Along-track minimum, maximum	0, 36.2	0, 9.0	0, −4.6

Range and Angles

The next simulation studies included improved angular observations. The first scenario, case 3a, consisted of adding 6 nights of optical tracking from Maui with a one observation pair per minute density to the 3 range station case described earlier. The orbital errors showed that the angular observations from only 6 nights of tracking was sufficient to solve for the 3 range station biases accurately. It appears that with less range biases to estimate, fewer angular observations are needed to resolve the range ambiguity.

Two additional cases were considered where the tracking was limited to the 3 range stations. This time, the range data were limited to the number of observations collected by the 3 stations during the 8 range station scenario; in essence, the range data from the other 5 stations is ignored. In addition to the 3 range stations, one case (case 3b) included two optical sites (Albuquerque and Maui) and one (case 3c) included a single optical sensor (Maui). Table 16 contains the orbital errors for all 3 range station plus angles cases. Table 16 shows that accurate orbits are possible when combining a single optical sensor with fewer range stations.

Single-Site Range and Angles

With the success of the 3 range station plus angles cases, single-site range and angles orbit determination for geosynchronous satellites was investigated. The scenario included a single radio transponder, station 1, and optical sensor both located in Hawaii. Three simulations were performed varying the amount of range data available; all three cases included 6 nights of optical tracking spanning 8 h per night and collecting an observation pair every minute. The first case, case 4a, collected only the 4 passes (approximately 120 observations) present in the 8 range station scenario. The second case, case 4b, combined the station 1 and 2 passes to collect 7 passes (approximately 210 observations). The third case, case 4c, lengthened the 7 passes to collect a total 775 range observations (still less than the 8-station case). In all cases, a bias was added to the range data and estimated in the differential corrections.

Table 17 contains the orbital errors for the 3 single-site cases. Figure 8 plots the orbital errors for case 4c. From Fig. 8 and Table 17, it can be seen that very accurate orbits can be produced from the single-site range and angles scenario. Additional range data help reduce the radial drift and along-track runoff present in some of the range and angles cases. In Table 17, cases 4a and 4b clearly contain some along-track runoff.

Table 18 Angles-only orbit errors

	Difference from truth, m	
Property	2 Optical sensors	1 Optical sensor
Radial periodics	3.2	12.1
Cross-track periodics	0.8	6.7
Along-track minimum, maximum	0, 14.5	0, 56

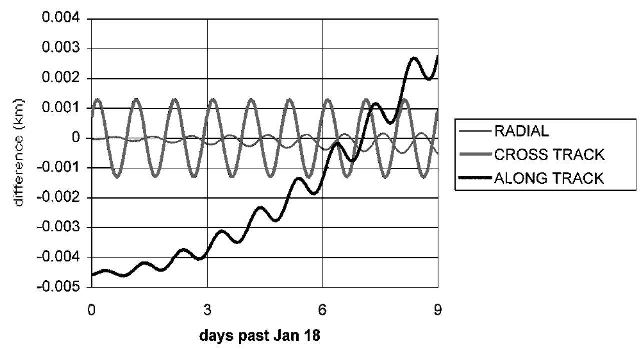

Fig. 8 Single-site range and angles orbit determination errors.

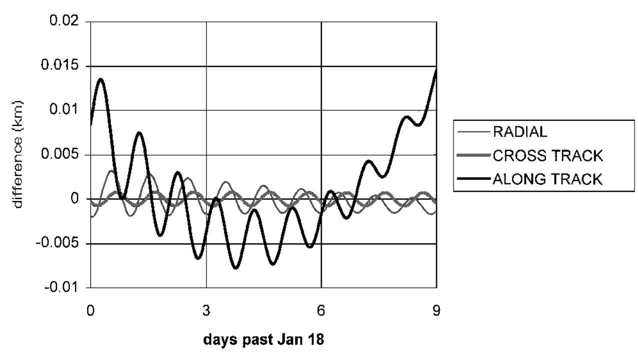

Fig. 9 Errors of 2-station angles-only orbit determination.

Angles Only

The next scenario was designed to determine the potential of angles-only geosynchronous orbit determination. Two cases were investigated. The first reflected the primary tracking scenario of optical sites in Albuquerque and Maui, each tracking 6 nights spanning the entire 9-day fit span and collecting an observation pair every minute. The second case only utilized the optical sensor in Albuquerque (single-site angles-only geosynchronous orbit determination). To strengthen the single-site result, a seventh day of tracking was added on 18 January. Table 18 contains the orbital errors for the angles-only cases. Figure 9 plots the orbital errors for the two-sensor solution.

For angles-only geosynchronous orbit determination to be successful, fairly long fit spans and a wide temporal data distribution are required. Without range data, the angular observations have a difficult time solving for the radial component of the satellite position in much the same way range-only solutions have difficulty solving for the along-track component. This radial uncertainty leads to large along-track runoff and very poor predictions. Through longer fit spans and data distributions, the period of the satellite helps define its altitude. However, the observation distribution from optical sensors is weather dependent, which makes angles-only orbit determination unreliable. Note that the two-optical-sensor case can theoretically provide better fit accuracy than the 8-station range-only case.

Conclusions

The potential of improved angular observations is demonstrated. For geosynchronous orbit determination scenarios involving extensive range tracking, it was shown the orbit solution is limited by the knowledge of the station range biases or the ability to solve for those biases. With the real data case presented, the range-only orbit error was on the order of 30 m with the potential to be much worse for lower inclinations. When improved angular observations were added in simulation studies, the uncertainty in the range bias solutions was greatly reduced, and the orbit solution errors dropped to the meter level. The angular observations allowed for the range biases to be accurately estimated even for low-inclination orbits. For the 8 range station case presented, two optical sensors are desirable to reduce the orbital errors; however, desired density of the angular observations allows the optical stations to track two geosynchronous satellites. Thus, two geosynchronous satellites can share two optical sensors and maintain greater orbit accuracy than each satellite utilizing a single optical sensor independently.

When the number of range stations is reduced, the number of angular observations required to bring the orbital errors to the meter level is also reduced. For a 3 range station case, studies show that a single optical sensor provides enough information to solve accurately for the 3 range station biases and reduce the orbital errors greatly from the range-only cases. Furthermore, it was shown that single-site range and angles geosynchronous orbit determination can produce orbits on the meter level. Note that, in simulation, a single range and optical station can produce greater accuracy orbits than an extensive 8 range station tracking network.

Angles-only geosynchronous orbit determination cases demonstrated the ability to produce accurate orbits (on the level of 10 m), but the angles-only scenario requires a wide temporal observation distribution to capture the radial component of the satellite position. Because the optical sensors are limited by weather conditions, the required observation distribution to provide a desired level of accuracy may not always be available.

A major assumption in all of the simulation studies was that the angular data contained no biases. It is believed that biases in the optical sensors should be well determined because the sensors can track objects with well-known positions, such as reference stars or GPS satellites. It is unclear to what level the biases will be known. The range and angles orbit determination cases will, thus, be limited by the accuracy of the optical sensor calibration. If the optical sensors are calibrated to the 0.05-arc-s level, this would result in an approximate 10 m of error in the orbit solutions.

Acknowledgments

Chris Sabol would like to thank the members of his doctoral committee: Robert Culp, David Spencer, Paul Cefola, George Born, Kenn Gold, C. Y. Chow, and Subhendu Datta. The authors would like to thank the Raven teams in Albuquerque (Richard Burns, Megan Bir, Scott Wallace, Eric Beck, Kim Luu, Tiffany Montague, Scott Carter, and Nate Devilbus) and Maui (Daron Nishimoto and Paul Sydney). The authors would also like to thank the people and organizations that provided us with the radio transponder data and information that allowed us to produce these studies. Others who have aided this study include David L. Harris, Patrick Wallace, Dave Monet, and Craig McLaughlin. Special thanks to Kim Luu for her help with this manuscript.

References

[1] Wallace, S., Sabol, C., Carter, S., "Use of the Raven Optical Sensor for Deep Space Orbit Determination," *Advances in the Astronautical Sciences*, Vol. 97, No. 2, 1998, pp. 1603–1621.

[2] Sabol, C., "A Role for Improved Angular Observations in Geosyncrhonous Orbit Determination," Ph.D. Dissertation, Dept. of Aerospace Engineering Sciences, Univ. of Colorado, Boulder, CO, Aug. 1998.

[3] Sabol, C., Burns, R., and Wallace, S., "Analysis of the Telstar-401/GOES-10 Close Approach Using the Raven Telescope," *Advances in the Astronautical Sciences*, Vol. 99, No. 1, 1998, pp. 317–332.

[4] Marshall, J. A., Lemoine, F. G., Luthcke, S. B., Chan, J. C., Cox, C. M., and Rowton, S. C., "Precision Orbit Determination and Gravity Field Improvement Derived from TDRSS," *Advances in the Astronautical Sciences*, Vol. 90, No. 1, 1996, pp. 183–200.

[5] Haines, B. J., Lichten, S. M., Srinivasan, J. M., Kelecy, T. M., and LaMance, J. W., "GPS-Like Tracking of Geosynchronous Satellites: Orbit Determination Results for TDRS and INMARSAT," *Advances in the Astronautical Sciences*, Vol. 90, No. 2, 1996, pp. 1225–1242.

[6] "Goddard Trajectory Determination System User's Guide, Revision 2-Update 6," Computer Sciences Corp., CSC/SD-85/6738R2UD6, Greenbelt, MD, Aug. 1993.